

Notes 4.5

Warmup – State the domain and the vertical asymptote for each given equation.

1.  $y = \frac{x-2}{(x-2)(x+5)}$

VA:  $x = 2, -5$

Domain:

$x \neq 2, -5$

2.  $y = \frac{x+6}{(x-4)(x+6)}$

VA:  $x = 4, -6$

Domain:

$x \neq 4, -6$

3.  $y = \frac{(x-7)(x+10)}{(x+10)(x-3)(x-7)}$

VA:  $x = -10, 3, 7$

Domain:

$x \neq -10, 3, 7$

Investigation

Think about how fractions are reduced? Apply the same rules to the three equations from the warmup to simplify each equation.

a.  $y = \frac{\cancel{x-2}}{(x-2)(x+5)}$

$y = \frac{1}{x+5}$

b.  $y = \frac{\cancel{x+6}}{(x-4)(x+6)}$

$y = \frac{1}{x-4}$

c.  $y = \frac{\cancel{(x-7)}(x+10)}{(x+10)(x-3)\cancel{(x-7)}}$

$y = \frac{1}{x-3}$

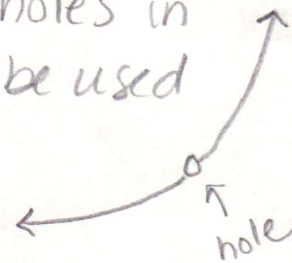
Each of these equations only has one vertical asymptote, which asymptote do you think is the one you would use to graph?

VA:  $x = -5$

VA:  $x = 4$

VA:  $x = 3$

The asymptotes that are not used for graphing must be included by *being holes in the graph because those values cannot be used in the original equations*



Practice simplifying these rational expressions:

d.  $\frac{2(x-4)}{6x(x-4)}$

$\frac{2}{6x}$

$\frac{1}{3x}$

e.  $\frac{x^2+3x-28}{x^2-49}$

$\frac{\cancel{(x+7)}(x-4)}{\cancel{(x+7)}(x-7)}$

$\frac{x-4}{x-7}$

f.  $\frac{x(x^2-x-42)}{2x^2-20x+42}$   $2(x^2-10x+21)$

$\frac{x\cancel{(x-7)}(x+6)}{2\cancel{(x-7)}(x-3)}$

$\frac{x(x+6)}{2(x-3)}$

## Multiplying and dividing rational expressions

Rational expressions are fractions, to multiply rational expressions *multiply straight across, then reduce*

g.  $\frac{x^2+7x-8}{x+8} \cdot \frac{x+5}{9x-9}$

factor  
cancel  
multiply

$$\frac{\cancel{(x+8)}\cancel{(x-1)}}{\cancel{(x+8)}} \cdot \frac{x+5}{9\cancel{(x-1)}}$$

$$\boxed{\frac{x+5}{9}}$$

h.  $\frac{10x^2-20x}{40x^3-80x^2} \cdot \frac{16x^3+80x^2}{6x+30}$

$$\frac{10x\cancel{(x-2)}}{40x^2\cancel{(x-2)}} \cdot \frac{16x^2\cancel{(x+5)}}{6\cancel{(x+5)}}$$

$$\frac{10x}{40x^2} \cdot \frac{16x^2}{6}$$

$$\frac{x}{4} \cdot \frac{8}{3}$$

$$\boxed{\frac{2x}{3}}$$

i.  $\frac{x+7}{7x+35} \cdot \frac{x^2-3x-40}{x-8}$

$$\frac{x+7}{7\cancel{(x+5)}} \cdot \frac{\cancel{(x-8)}\cancel{(x+5)}}{\cancel{x-8}}$$

$$\boxed{\frac{x+7}{7}}$$

j.  $\frac{x^2+6x+5}{x+1} \cdot \frac{x^2+11x+28}{x^2+9x+20}$

$$\frac{\cancel{(x+5)}\cancel{(x+1)}}{\cancel{x+1}} \cdot \frac{\cancel{(x+7)}\cancel{(x+4)}}{\cancel{(x+5)}\cancel{(x+4)}}$$

$$\boxed{x+7}$$

To divide rational expressions take the reciprocal of the second fraction, then multiply

k.  $\frac{(x-7)(x+8)}{(x+8)(x-10)} \div \frac{1}{x-10} \rightarrow$

$$\frac{(x-7)\cancel{(x+8)}}{\cancel{(x+8)}\cancel{(x-10)}} \cdot \frac{\cancel{x-10}}{1}$$

$$\boxed{x-7}$$

l.  $\frac{x^2-16}{-1(x-9)} \div \frac{x^2+14x+40}{x^2+x-90}$

$$\frac{\cancel{(x+4)}\cancel{(x-4)}}{-1\cancel{(x-9)}} \cdot \frac{\cancel{(x+10)}\cancel{(x-9)}}{\cancel{(x+10)}\cancel{(x+4)}}$$

$$\boxed{-1(x-4)}$$

$$m. \frac{7x^2}{7x^3+56x^2} \div \frac{2}{x^2+7x-8}$$

$$\frac{\cancel{7x^2}}{\cancel{7x^2}(x+8)} \cdot \frac{(x+8)(x-1)}{2}$$

$$\boxed{\frac{x-1}{2}}$$

$$n. \frac{x^2+2x-3}{x^2-3x-10} \div \frac{x-1}{x+5}$$

$$\frac{(x+3)(\cancel{x-1})}{(x-5)(x+2)} \cdot \frac{x+5}{\cancel{x-1}}$$

$$\frac{(x+3)(x+5)}{(x-5)(x+2)}$$

Practice:

$$o. \frac{93}{21x} \cdot \frac{34x}{51x}$$

$$\frac{31}{7x} \cdot \frac{2}{3}$$

$$\boxed{\frac{62}{21x}}$$

$$p. \frac{x-4}{x^2-2x-8} \div \frac{1}{x-5}$$

$$\frac{x-4}{(x-4)(x+2)} \cdot \frac{x-5}{1}$$

$$\boxed{\frac{x-5}{x+2}}$$

$$q. \frac{x^2-10x+25}{10x-100} \cdot \frac{x-10}{9x-45}$$

$$\frac{(x-5)(x-5)}{10(x-10)} \cdot \frac{x-10}{9(x-5)}$$

$$\boxed{\frac{x-5}{90}}$$

$$r. \frac{6x+27}{18x^2+36x} \div \frac{16x+72}{2x+4}$$

$$\frac{3(2x+9)}{18x(x+2)} \cdot \frac{2(x+2)}{8(2x+9)}$$

$$\frac{\cancel{3} \cdot \cancel{2}}{6 \cdot 18x \cdot \cancel{8} \cdot \cancel{4}}$$

$$\boxed{\frac{1}{24x}}$$